University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

## ASSIGNMENT 7

**Reading:**
105 Notes 8.1-8.3, 6.1-6.2 (again).
Hand & Finch 4.7, 5.1.

**1.**
[This a (hopefully clearer) version of Hand & Finch 4.17, "tetherball".] A mass $m$ is attached to a weightless string that initially has a length $s_0$. The other end of the string is attached to a post of radius $a$. Neglect the effect of gravity. Suppose that the mass is set into motion. It is given an initial velocity of magnitude $v_0$ directed so that the string remains taut. The string wraps itself around the post, causing the mass to spiral inward toward it.

(**a**)
Write the Lagrangian in terms of $\dot{x}$ and $\dot{y}$, the cartesian velocity components of the mass. Is there a potential energy term?

(**b**)
Use as generalized coordinates $s(t)$, the length of the part of the string that is not yet in contact with the post, and $\psi(t)$, the azimuthal angle at which the string barely fails to make contact with the post. Express $\dot{x}$ and $\dot{y}$ in terms of these generalized coordinates and their time derivatives.

(**c**)
Write a (constraint) equation relating $\dot{s}$ to $\dot{\psi}$. Use it to greatly simplify your answers for (**b**). Rewrite the Lagrangian using $s$ as the only generalized coordinate.

(**d**)
Use the Euler-Lagrange equation to obtain an equation of motion for $s$. (You don't need to solve it.)

(**e**)
Since the Lagrangian has no explicit time dependence, and it depends quadratically on $\dot{s}$, the total energy is conserved. Write an equation setting the initial energy (expressed in terms of $v_0$) equal to the energy at an arbitrary value of $s$ (expressed in terms of $s$ and $\dot{s}$).

(**f**)
Use this equation to express $dt$ in terms of $ds$ multiplied by a function of $s$. Integrate it to solve for the time $T$ that elapses before the mass hits the post. You should obtain the simple result

$$T = \frac{s_0^2}{2av_0} \ .$$

(**g**)
Is the angular momentum of the mass about the *axis* of the post conserved in this problem? Why or why not?

**2.**
Hand & Finch 4.19.

**3.**
Hand & Finch 4.21.

**4.**
Consider a particle of mass $m$ that is constrained to move on the surface of a paraboloid whose equation (in cylindical coordinates) is $r^2 = 4az$. If the particle is subject to a gravitational force $-mg\hat{\mathbf{z}}$, show that the frequency of small oscillations about a circular orbit with radius $\rho = \sqrt{4az_0}$ is

$$\omega = \sqrt{\frac{2g}{a + z_0.}}$$

**5.**
An orbit that is almost circular can be considered to be a circular orbit to which a small perturbation has been applied. Take $\rho$ to be the (unperturbed) circular orbit radius and define

$$g(r) = \frac{1}{\mu}\frac{\partial U(r)}{\partial r} \ ,$$

where $\mu$ is the reduced mass and $U$ is an arbitrary potential. Set the radius $r = \rho + x$, where $x$ is a small perturbation.

**(a)**

Starting from the differential equation for $r$ and using the fact that the angular momentum $l$ is constant, substitute $r = \rho + x$. Retaining terms only to first order in $x$, Taylor expand $g(r)$ about the point $r = \rho$, and show that $x$ satisfies the differential equation

$$\ddot{x} + \left[\frac{3g(\rho)}{\rho} + g'(\rho)\right] x = 0 \ ,$$

where $g'(\rho)$ is $dg/dr$ evaluated at $r = \rho$.

**(b)**

Taking the force law to be $F(r) = -kr^{-n}$, where $n$ is an integer, show that the angle between two successive values of $r = r_{\text{max}}$ (the "apsidal angle") is $2\pi/\sqrt{3-n}$. Thus, if $n > -6$, show that in general a closed orbit will result only for the harmonic oscillator force and the inverse square law force.

**6.**

Consider the motion of a particle in a central force field $F = -k/r^2 + C/r^3$.

**(a)**

Show that the equation of the orbit can be put in the form

$$\frac{1}{r} = \frac{1 + \epsilon \cos \alpha\theta}{a(1-\epsilon^2)} \ ,$$

which is an ellipse for $\alpha = 1$, but is a *precessing* ellipse for $\alpha \neq 1$.

**(b)**

The precessing motion may be described in terms of the *rate of precession of the perihelion*, where the term perihelion is used (loosely) to denote *any* of the turning points of the orbit. Derive an approximate expression for the rate of precession when $\alpha$ is close to unity, in terms of the dimensionless quantity $\eta = C/ka$.

**(c)**

The ratio $\eta$ is a measure of the strength of the perturbing inverse cube term relative to the main inverse square term of the force. Show that the rate of precession of Mercury's perihelion ($40''$ of arc per century) could be accounted for *classically*, if $\eta = 1.42 \times 10^{-7}$. [Mercury's period and eccentricity are 0.24y and 0.206, respectively.]

**7.**

A He nucleus with velocity $v = 0.05c$ is normally incident on an Au foil that is 1 micron ($1 \times 10^{-6}$ m) thick. What is the probability that it will scatter into the backward hemisphere, *i.e.* bounce off the foil? (Please supply a *number*.)

**8.**

Calculate the differential cross section $d\sigma/d\Omega$ and the total cross section $\sigma_T$ for the elastic scattering of a point particle from an impenetrable sphere; *i.e.*, the potential is given by $U(r) = 0,\ r > a$; $U(r) = \infty,\ r < a$.